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Reply to "Comment on 'Calculation of ionization balance and electrical conductivity in nonideal aluminum plasma' "

Deok-Kyu Kim* and Inho Kim

Agency for Defense Development, P.O. Box 35-5, Yuseong, Daejeon 305-600, Korea

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We explain that the classical integral expression of the average electron-ion momentum transfer cross section is of limited applicability to dense plasmas without correcting the cutoff screening radius approximation, and that the Zollweg-Liebermann model appears practically useful to reproduce the experimental data with mathematical simplicity.

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Zaghloul claims [1] that Zollweg and Liebermann's fitting formula [2] of the electron-ion collision cross section is unacceptably inaccurate and should not be used to calculate the transport properties of nonideal plasmas [3], because it fails to recover the exact values of the classical analytic expression of the energy-averaged electron-ion momentum transfer cross-section integral. Instead, Zaghloul proposed a new analytic formula that accurately fits the classical integral expression.

As Zaghloul indicated in the Comment, the Zollweg-Liebermann fit indeed yields much higher cross-section values than predicted by the classical cross-section integral expression by $\sim$100% in its proposed range of applicability. However, it should be noted that this classical integral expression has been derived based on the assumption of finite cutoff screening radius defined by the Debye shielding length λ_D in a binary collision model, and can only serve as a rough estimation in most practical cases since its validity is limited by a restrictive condition $\Lambda \gg 1$, where Λ is the Coulomb logarithm defined as the ratio of the Debye length to the average impact parameter $\overline{b_0}$ [4]. In particular, when the plasma density is high, this classical expression tends to yield substantially underestimated values of electron-ion collision cross section because the Debye length rapidly decreases and the number of electrons within a Debye sphere becomes too small to shield out the ion charge effectively. Therefore, in order to obtain physically meaningful cross-section values in a dense plasma regime, one should take into account the effect of enhanced screening radius.

Some authors have attempted to present detailed descriptions of the effective screening radius or effective collision frequency in nonideal plasmas [5,6], but, for simplicity, our previous calculation of the electrical conductivity utilized the fitting formula $\ln(1+1.4\Lambda^2)^{1/2}$ given by Zollweg and Liebermann. Although a question may remain about the derivation of this formula, it is interesting to note that the fitting factor used in the Zollweg-Liebermann model is found to be consistent with the physical consideration of enhanced screening radius in nonideal plasmas, though its available parameter range is restricted to a narrow region. In addition, their modifying the Coulomb logarithm to have its minimum value limited by the interionic distance $\lambda_+ = \left(\frac{4}{3}\pi n_+\right)^{-1/3}$ permits a description of plasmas at extremely high density. We also noticed that Zollweg and Liebermann's original paper showed a reasonable agreement of their calculations with experimental data available at that time. Comparisons with other theoretical models revealed no unacceptably large order-of-magnitude discrepancies [5]. Moreover, as shown

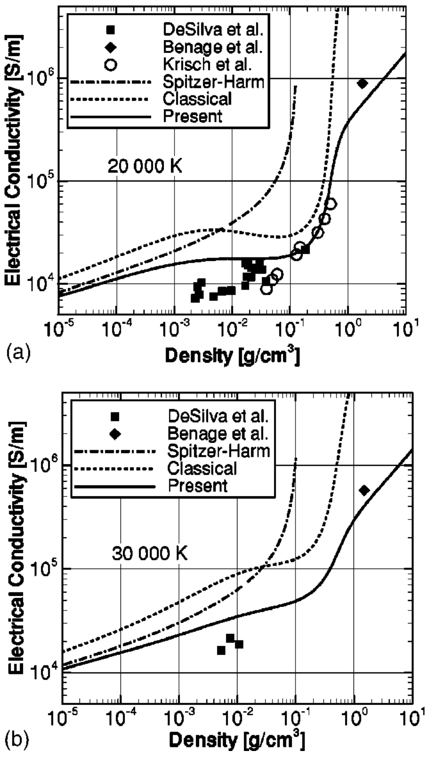

FIG. 1. Electrical conductivities calculated with Spitzer-Harm approximation, classical cross-section integral, and Zollweg-Liebermann fitting formula in comparison with experimental data measured by DeSilva *et al.* and Benage *et al.* at temperatures of (a) 20 000 K and (b) 30 000 K.

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*Electronic address: dkyukim@add.re.kr

from our calculated results compared with the recent capillary wire discharge experiments [7–9], the Zollweg-Liebermann model appears practically useful to represent the measured data in dense plasma conditions.

Here, for a clear understanding of the different physical model features, we have evaluated electrical conductivities of aluminum plasma using the numerically integrated values of the classical cross-section integral formula with the neutral atoms taken into account, and we compared them with our previous calculations, which employed the Zollweg-Liebermann fit, and the experimentally measured data in the same parameter range. The results are presented in Fig. 1 for a sufficiently high degree of ionization at $T=$(a) 20 000 K and (b) 30 000 K. As anticipated, the electrical conductivities evaluated using the numerically integrated classical cross-section integral without correcting the cutoff screening radius appear essentially higher than the results computed with the Zollweg-Liebermann model at high temperatures, leading to much greater discrepancies with measured data. The classical model also gives larger conductivities than the Spitzer-Härm values in a low-density plasma regime.

In conclusion, the analytic formula proposed by Zaghloul may provide an efficient way to accurately evaluate the average electron-ion cross-section integral under the regime of the classical plasma approximation. For calculation of plasma transport coefficients at higher densities, however, it is required to include an appropriate account of the effective screening radius. Although the Zollweg-Liebermann model might seem oversimplified or even poorly grounded in a sense, we regard this model as being of practical advantage in calculating the electrical conductivity of nonideal plasma since it reasonably reproduces the experimental data over a large parameter domain with negligible computational efforts.

It is clear that a more accurate physical model capable of describing nonideal plasmas is strongly required to replace simplified ones such as was used in our calculation, and if the physical considerations of dense plasma effects mentioned above can be implemented in a proper manner, the analytic formula proposed by Zaghloul would be of practical applicability with a guaranteed accuracy.

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